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Periodicity in spatial data and geostatistical models: autocorrelation between patches

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Several recent studies in landscape ecology have found periodicity in correlograms or semi-variograms calculated, for instance, from spatial data of soils, forests, or animal populations. Some of the studies interpreted this as an indication of regular or periodic landscape patterns. This interpretation is in disagreement with other studies that doubt whether such analysis is valid. The objective of our study was to explore the relationship between periodicity in landscape patterns and geostatistical models. We were especially interested in the validity of the assumption that periodicity in geostatistical models indicates periodicity in landscape pattern, and whether the former can characterize frequency and magnitude of the latter. We created maps containing various periodic spatial patterns, derived correlograms from these, and examined periodicity in the correlograms. We also created non-regular maps that we suspected would cause periodicity in correlograms. Our results demonstrate that a) various periodic spatial patterns produce periodicity in correlograms derived from them, b) the distance-lags at which correlograms peak correspond to the average distances between patch centers, c) periodicity is strongest when the diameter of patches is equal to the distance between patch edges, d) periodicity in omni-directional correlograms of complex spatial patterns (such as checkerboards) are combinations of several waves because inter-patch distances differ with direction; multiple directional correlograms can decompose such complexity, and e) periodicity in correlograms can also be caused when the number of patches in a study site is small. These results highlight that correlograms can be used to detect and describe regular spatial patterns. However, it is crucial to ensure that the assumption of stationarity is not violated, i.e., that the study area contains a sufficiently large number of patches to avoid incorrect conclusions.

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The importance of spatial patterns for ecological processes was recognized early (Leopold 1933, Watt 1947) but remains a challenge and an active area of research in ecology (Levin 1992). Various tools have been proposed to quantify patterns, but development of these tools is far from complete (Gustafson 1998). Geostatistical tools that analyze spatial autocorrelation were proposed as early as 1926 (Langsaeter, cited in Rossi et al. 1992), but it was not until recently that they have been widely used in ecology (Burrough 1983a, b, Legendre and Fortin 1989, Turner et al. 1991, Rossi et al. 1992, Liebhold et al. 1993, Horne and Schneider 1995, Koenig 1999). Concurrently, landscape ecology developed from early roots (Troll 1939) into a sub-discipline of ecology that generated wide interest in spatial patterns across landscapes (Forman and Godron 1986, Turner 1989, Kareiva and Wennegren 1995). The application of variograms and correlograms to landscapes with periodic spatial patterns, such as equally distant ridges, soon followed these developments (Lobo et al. 1998).

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Periodic landscape patterns can result in periodic patterns in geostatistical models (Sokal 1979, Legendre and Fortin 1989, Chou 1995). Indeed, a number of studies recognized such spatial periodicity in geostatistical models and described these patterns as "cyclic" (Burrough 1987, Gustafson 1998), "periodic" (Pierson and Wight 1991, Dammer et al. 1997), or "wave semivariograms" (Cressie 1993, Oliver et al. 1997). This phenomenon has been found in a wide range of scientific fields but no common terminology exists. Terms that are applied include "second-order autoregressive correlation functions" in wind-fields (Thiébaux 1985), "periodic variograms" observed in remotely sensed data of forests (Curran 1988, Cohen et al. 1990) and near-surface soil temperature on sagebrush rangeland (Pierson and Wight 1991), "a cyclical component" in semi-variograms of soil-water content and hydraulic conductivity (Ciollaro and Romano 1995), and "sinusoidal patterns of semivariance" identified in spatial patterns of moose-forest-soil interactions (Pastor et al. 1998). In the earth sciences, scientists speak of "hole effect variograms", caused, for instance, by discrete lenses of ore in a deposit with equal spacing between adjacent lenses (Isaaks and Srivastava 1989), or "periodic hole-effect (hole-sinusoidal) covariance models" (Ababou et al. 1994). Legendre and Fortin (1989) described "peaks and troughs" in correlograms derived from artificial data sets. Attempts to fit models to such variograms were based on sinusoidal functions (Thiébaux 1985, Cressie 1993, Shapiro and Botha 1991, Oliver et al. 1997, Lobo et al. 1998) also referred to as a "harmonic oscillator model" (Pastor et al. 1998).

There has been agreement in the literature that periodicity in spatial patterns will lead to periodicity in the geostatistical model. Chou (1995) analyzed artificial landscapes containing periodic spatial patterns and suggested the use of wavelength and amplitude of a periodic correlogram to reveal "the topological structure of a complex spatial pattern" (Chou 1995, p. 365). Earlier, Legendre and Fortin (1989) pointed out that the lag distance of peaks in a correlogram corresponds to the distance between patches. Taking this logic one step further, Pastor et al. (1998) postulated that periodicity in a geostatistical model indicates "a landscape in which homogeneous patterns are regularly arranged" (Pastor et al. 1998, p. 417). However, such an interpretation of periodicity in variograms was questioned by Rossi et al. (1992), who showed that such patterns could be caused by differences between local means and variances, and thus a violation of the assumption of stationarity.

Clearly, there is little agreement about the interpretation of apparent periodicity in geostatistical models within ecology. This is unfortunate for two reasons: first, the increasing use of geostatistics in ecology makes it likely that such patterns will be found again; clear guidelines about their interpretation could assist comparison between studies and prevent incorrect conclusions. Second, regular spatial patterns are of great interest for ecology. Such patterns have been found in the results of cellular models, especially when modeling dispersal or spread of disturbances. Examples include studies of insect population dynamics (Hassell et al. 1991), forest dieback (Iwasa et al. 1991, Jeltsch and Wissel 1994), and fungi spread (Halley et al. 1994). Geostatistics could potentially play an important role in detecting and analyzing such patterns in real landscapes if the conclusions of Pastor et al. (1998) were found to be generally true.

The objective of our study was to explore the relationship between periodicity in landscape patterns and geostatistical models and to clarify the contradicting conclusions presented in recent studies (e.g., Rossi et al. 1992, Pastor et al. 1998). We were especially interested in the validity of the assumption that periodicity in geostatistical models indicates periodicity in landscape pattern, and if the former can characterize frequency and magnitude of the latter.

Methods

Our approach followed that of Sokal (1979), Legendre and Fortin (1989), Chou (1993, 1995), and Meisel and Turner (1998): maps were generated containing periodic spatial patterns, and geostatistical models were derived from these. Maps were rectangular and 150×150 data points in size, with equal distance units between points. Each point was assigned a presence-absence value according to the spatial patterns under consideration. Spatial patterns were created by allocating square patches on the map.

As a neutral model, we created maps of randomly located non-overlapping patches. We analyzed these and various forms of spatial periodicity (Fig. 1). Certain attributes of some of these patterns, such as distances between patches and patch size, were altered to reveal the effects of these attributes on geostatistical models. In addition, we also generated maps that did not contain spatial periodicity, but that we suspected to result in periodicity in the geostatistical models derived from them (Fig. 2).

All maps were analyzed using correlograms $\hat{\rho}(h)$ of the form

$$\hat{\rho}(h) = 1 - \frac{\hat{\gamma}(h)}{C(0)} \qquad (1)$$

where C(0) is the finite variance of the random field and $\hat{\gamma}(h)$ is the semivariogram for pairs of points separated by the distance h defined as

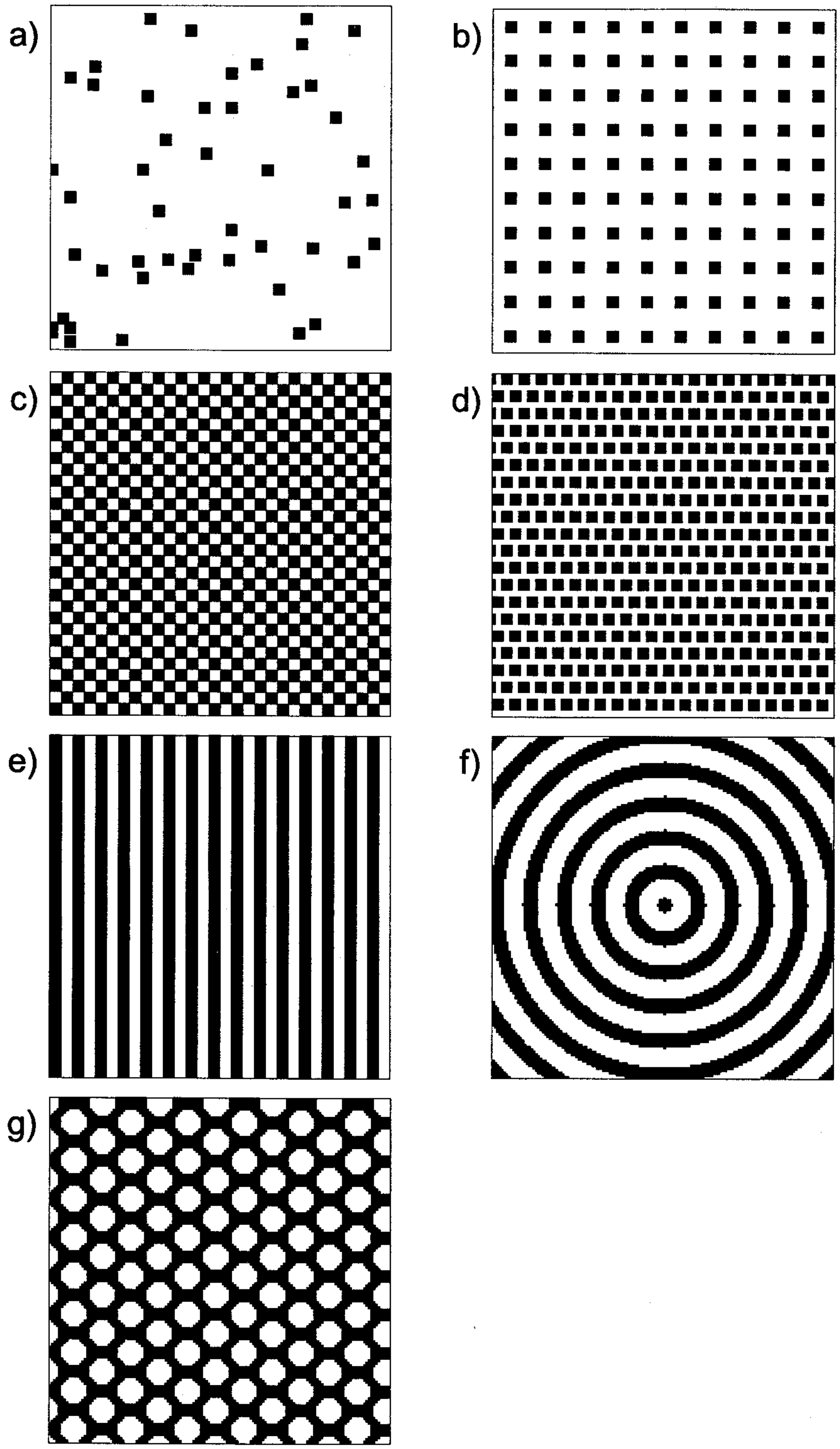

Fig. 1. Maps (150 × 150 distance units) depicting basic forms of spatial periodicity: a) randomly located pairs of patches (50 patches, each 5 × 5 distance units in size), b) grid (5 × 5 patches, 15 distance units between patch centers in horizontal and vertical direction), c) checkerboard (5 × 5 patches), d) triangular (5 × 5 patches, 8 distance units between centers of neighboring patches), e) ridges (5 distance units wide, 10 distance units between ridge centers), f) nested rings (6 distance units wide, 15 distance units between ring centers), and g) beehive (4 distance unit wide black ridges, 16 distance unit between ridge centers in vertical direction).

$$\hat{\gamma}(h) = \frac{1}{2|N(h)|} \sum_{N(h)} (z_i - z_j)^2 \qquad (2)$$

with N(h) being the set of all pairwise Euclidean distances $i - j = h$, $|N(h)|$ being the number of distinct distance pairs in N(h), and z_i and z_j being data values at the spatial locations i and j (Cressie 1993). We chose correlograms instead of semi-variograms for our analysis because of their robustness in the case that local means and variances vary (Rossi et al. 1992). We limited the computation of correlograms to up to half of the maximum distance in our maps to avoid edge effects. In a first step, we calculated omnidirectional correlograms from all maps. Second, we calculated directional correlograms for maps where neighboring patches in one direction are closer than in other directions (e.g., Fig. 1b, horizontal and vertical vs diagonal direction).

Results

Effects of different spatial patterns on correlograms

All omnidirectional correlograms derived from the maps presented in Fig. 1 show autocorrelation up to 5 distance units, which corresponds to the size of single patches (Fig. 3). This is most apparent in the correlogram for the randomly located patches for which autocorrelation drops to zero at larger distance classes (Fig. 3a). The correlograms derived from all other maps exhibit peaks and troughs in the shape of waves, thus showing some form of periodicity in the geostatistical model (Fig. 3b–g). The first positive peak of any given wave occurred at the average distance between the center of neighboring patches. For instance, the first peak in the correlogram for the grid (Fig. 3b) and the ridges (Fig. 3e), occurred at a distance of 15, compared to 18 in the case of the circles (Fig. 3f). Further peaks occurred roughly at multiples of the distance of the first peak, for instance, in the case of the correlogram for the grid at distances of 30 and 45 (Fig. 3b).

However, the smoothness of the periodicity in the correlograms varies and certain correlograms, for instance the one calculated for the grid (Fig. 3b), appeared noisy. The reason is that the center of neighboring patches in the horizontal and vertical direction of the grid (Fig. 1a) is only 15 distance units apart, whereas the distance is 21 along the diagonals. The directional correlograms showed the first peak of autocorrelation in horizontal and vertical directions at 15 distance units, whereas the first peak for the diagonal distances occurred at 21 distance units (Fig. 4). The noise in the wave of the omnidirectional correlogram is the result of the combination of two waves with different wavelenghts. It is also important to note that periodicity in correlograms does not necessarily have a sinusoidal shape as apparent in this example. The correlogram of complex spatial patterns such as the honeycomb (Figs 1g, 3g) is the combination of multiple distinct waves from correlograms calculated in the six major directions of periodicity in the spatial patterns (Fig. 5). This complex combination of different waves resulted in an omnidirectional correlogram with a decrease in the amplitude of the wave around distances of 50–80 followed by an increase at larger distances (Fig. 3g).

Effects of changing patch size and inter-patch distances on correlograms

The next step was to examine results from changed attributes of the artificial maps in the correlograms. Only changes for the grid patterns are presented (Fig. 1b); they are exemplary of changes in the other patterns. Periodicity in the correlograms changed considerably when patch size was altered but distance between patch centers remained constant (Fig. 6). Periodicity was most apparent when the diameter of patches was roughly equal to the gap between the patches (7×7 and 9×9 patches when patch centers are 15 distance units apart). Smaller and larger patches exhibited less apparent periodicity, a smaller amplitude between

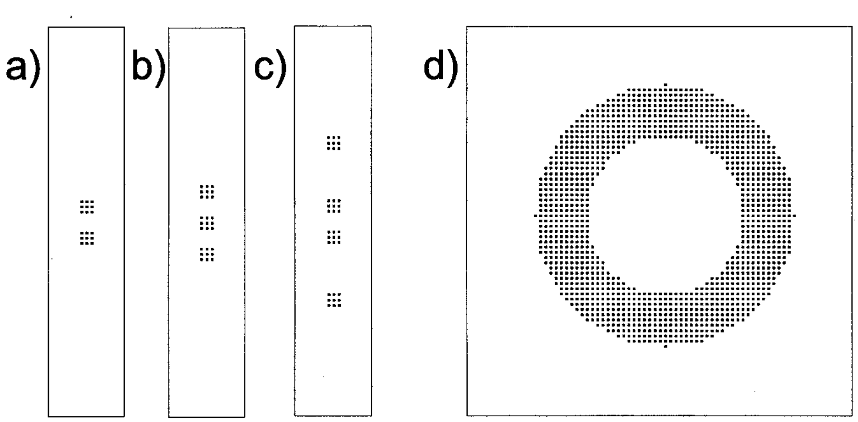

Fig. 2. Maps without spatial periodicity that are likely to cause periodicity in geostatistical models; transects containing a) 2 patches, b) 3 patches, c) 4 patches, and d) a single ring.

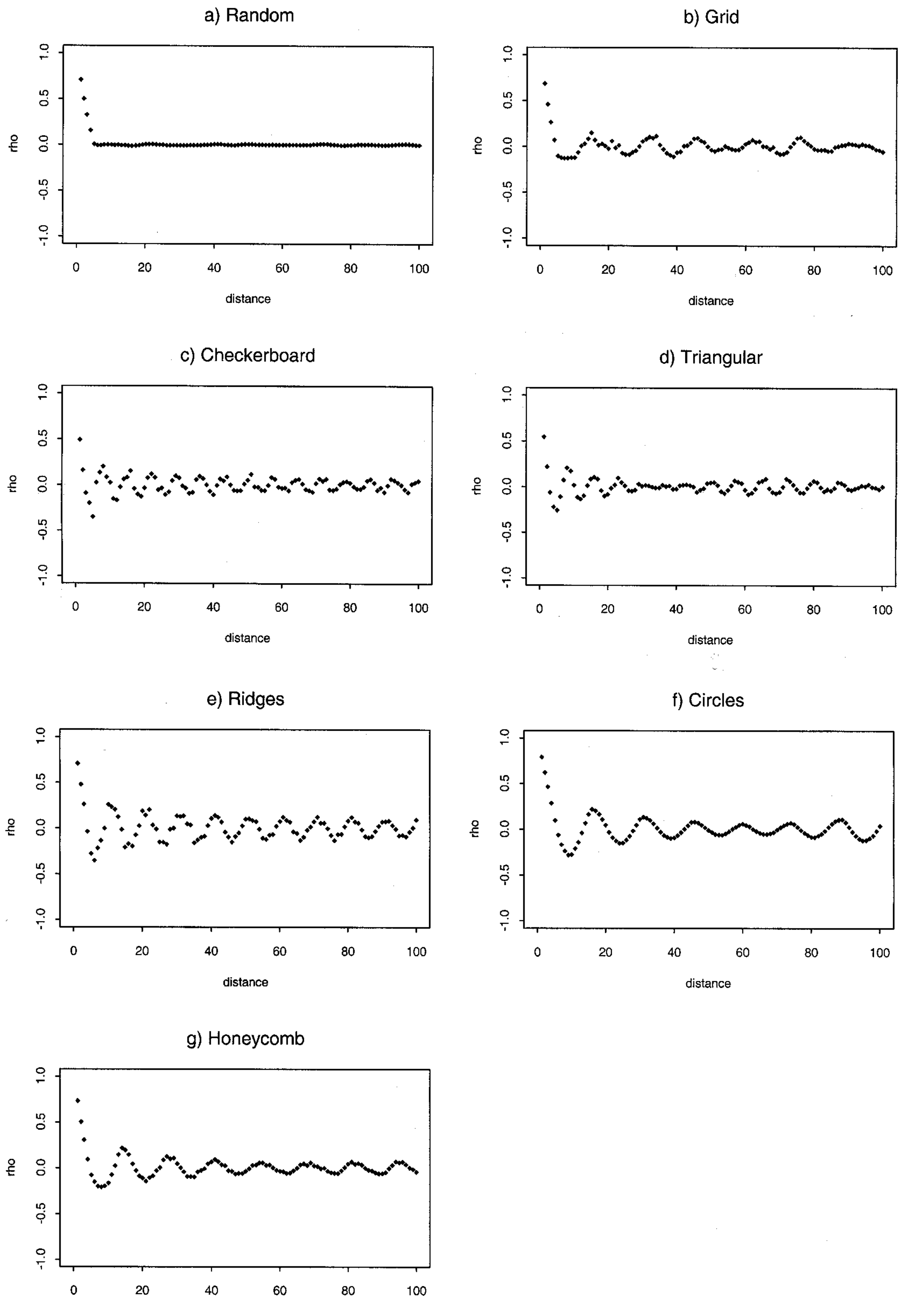

Fig. 3. Omnidirectional correlograms derived from the maps introduced in Fig. 1: a) randomly located pairs of patches, b) grid, c) checkerboard, d) triangular, e) ridges, f) nested rings, and g) beehive.

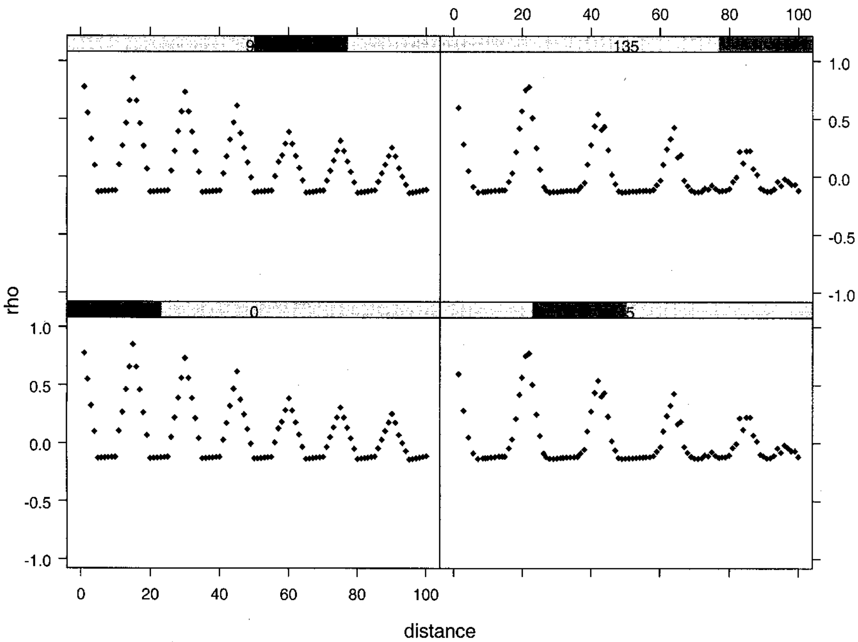

Fig. 4. Directional correlograms calculated for the grid (Fig. 1b) in the directions of 90°, 135°, 0°, and 45° (± 5°, 0° equal the top of the map).

peaks and troughs, and a different wave shape: the largest patches (13 × 13) appeared least sinusoidal. The location of the peaks and the wavelength remained constant and corresponded to the distance between patch centers.

The correlograms also changed when the size of the gap between patches was altered while patch size remained constant (Fig. 7). Unlike the previous example, the peak of the waves no longer occurred at fixed distances; as the distance between patch centers increased, the first peak of the wave occurred at a greater distance. Periodicity was most apparent up to the point where the size of the gap between patches (4 or 6) roughly equaled their size (5 × 5). Noise in the correlograms increased with larger gap sizes (Fig. 7).

Non-periodic spatial pattern and periodicity in correlograms

Last, we analyzed non-periodic spatial patterns that were suspected to result in periodicity in correlograms

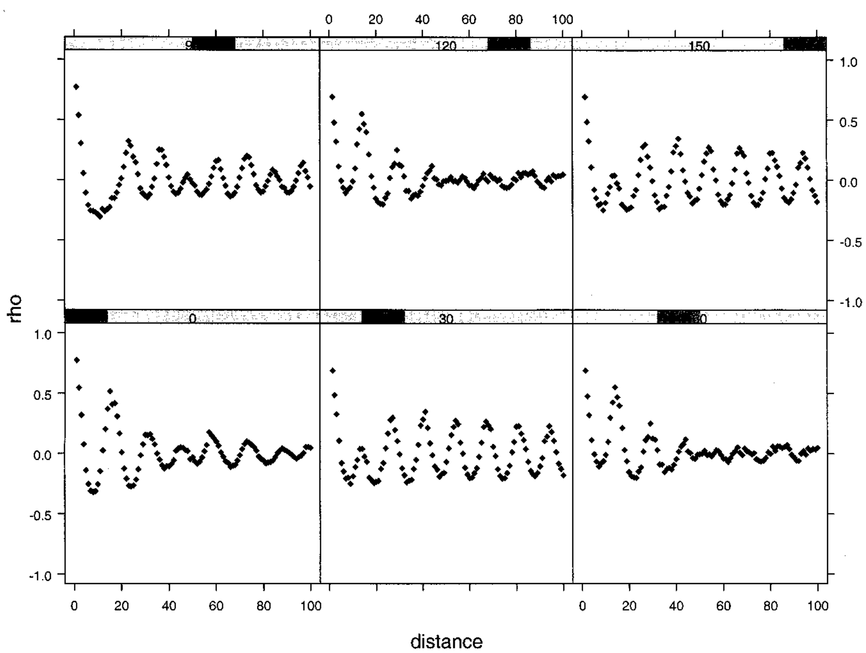

Fig. 5. Directional correlograms calculated for the honeycomb (Fig. 1g) in the directions of 90°, 120°, 150°, 0°, 30° and 60°.

derived from them (Fig. 2). Indeed, the resulting correlograms exhibited additional peaks corresponding to autocorrelation between patches (Fig. 8). The correlogram for two patches along a transect (Fig. 2a) showed one additional peak that corresponded to the distance between the two patches. Accordingly, a third and fourth patch located in the same direction cause further peaks in the correlogram when the distances between patch centers are equal, or multiples of the smallest distance. In the case of a single ring (Fig. 2c), the correlogram contained one additional peak that corresponded to the diameter of the ring.

Discussion and conclusion

It may be argued that none of our results are particularly surprising, and that many practitioners of geostatistics may have been able to correctly estimate the form of correlograms for most of our spatial patterns. Also, the "periodic" correlograms, i.e., the additional peaks in the correlograms for the maps without periodic spatial pattern, are by no means unexpected. However, there is continuing confusion in the recent ecological literature about the interpretation of periodicity in geostatistical models. Omnidirectional variograms with only one or two peaks have led to the conclusion that "one is moving through a landscape in which homogeneous patches are regularly arranged" (Pastor et al. 1998, p. 417). Our results suggest that more caution and additional analysis is needed when interpreting periodic patterns in geostatistical models.

Rossi et al. (1992, p. 305) pointed out correctly that "sinusoidal variation in the semi-variogram ... often ascribed to a periodicity in the data, ... was essentially due to differences between local means and variances". More simply, a study area with just a handful of patches is not suitable for revealing regular pattern, or,

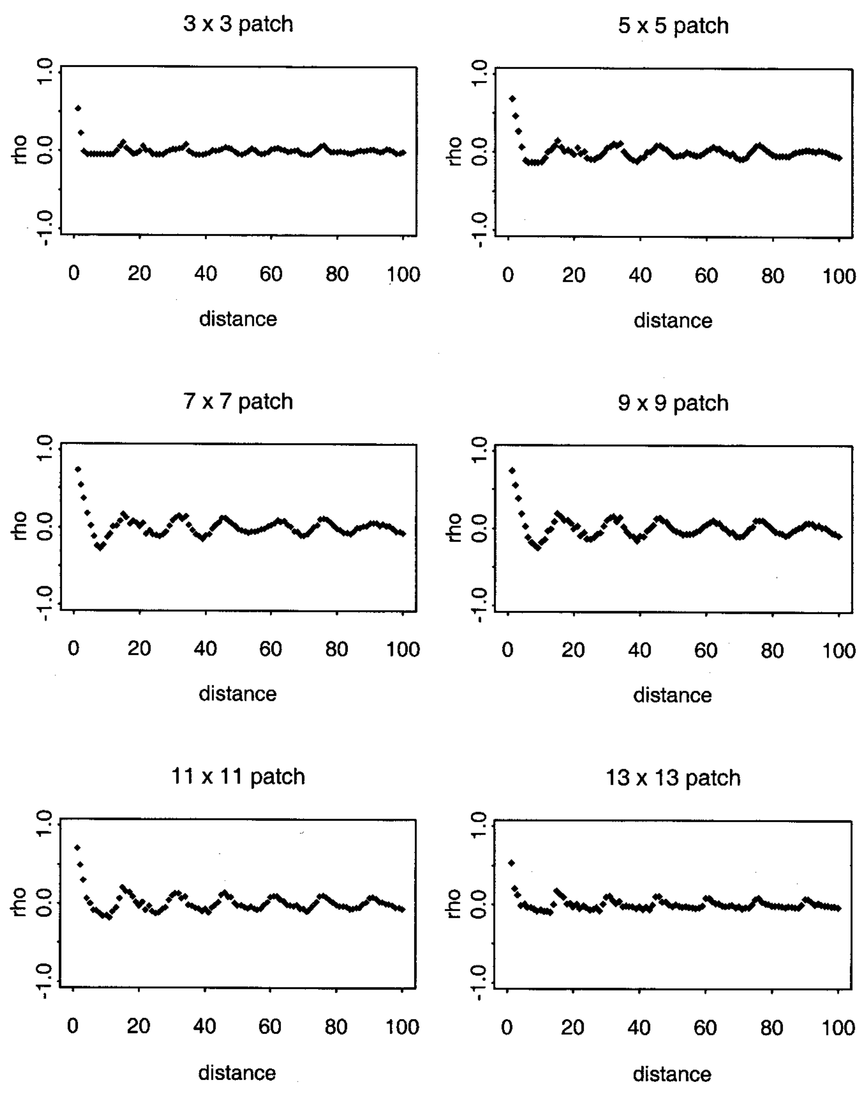

Fig. 6. Effect of changing patch size in a grid (Fig. 1b) with distance between patch centers being constant.

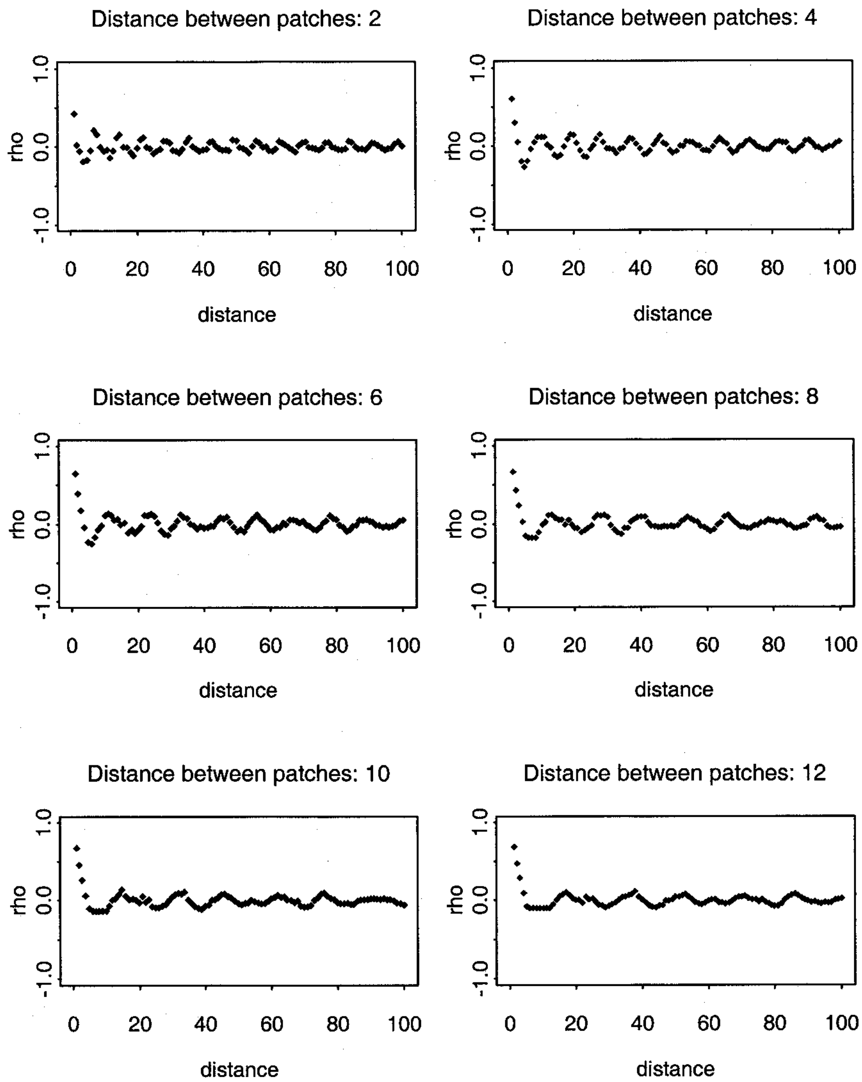

Fig. 7. Effect of changing distances between patch centers in a grid (Fig. 1b) with patch size being constant.

in the lingua of a geostatistician, stationarity is a requirement when calculating semi-variograms and correlograms. The analysis of one transect containing three sagebrush shrubs does not permit one to conclude that "sagebrush plants produced significant periodic spatial patterns in soil temperature across the landscape" (Pierson and Wight 1991, p. 12).

However, periodicity in correlograms can also reflect spatial periodicity as shown in this study; apparent periodicity in a correlogram does certainly warrant closer examination. We suggest that researchers first examine the original map for potential periodic spatial pattern. A good example was provided by Lobo et al. (1998), who visually identified three neighboring ridges in their study area. Second, omnidirectional correlograms most likely contain several different directional waves of different amplitude and wavelengths. Directional correlograms are better suited to examine spatial periodicity in detail. Third, other techniques, such as wavelet analysis (Li and Loehle 1995, 1996), spectral analysis (Ripley 1978), fourier analysis (Dammer et al. 1997), fractal models (Loehle and Li 1996), quadrat variance analysis (Dale and MacIsaac 1989, Dale 1990, Dale and Blundon 1990), and power spectra (Ciollaro and Romano 1995, Oliver et al. 1997), are available to identify spatial periodicity and should be employed when in doubt about their existence. Our results show that the interpretation of correlograms is not straightforward and that many different spatial patterns can result in similarly-shaped correlograms. The interpretation of geostatistical models alone is not sufficient for making assumptions about the nature of spatial periodicity in the underlying data.

We presented only correlograms in our results, because of their greater robustness in the case that local means and variances vary (Rossi et al. 1992), and because their significance can be tested (Legendre and Fortin 1989). Both of these studies advised use of correlograms rather than semi-variograms, or to employ both, when analyzing autocorrelated data. However, the close relationship between correlograms and semi-variograms makes our results also applicable to analysis of the latter. Also, for easier interpretation of the results, we presented only analysis of presence-absence data. However, periodicity in the spatial patterns of continuous variables also causes periodicity in resulting geostatistical models (Legendre and Fortin 1989).

Our artificial maps containing spatial periodicity are much more likely to result in periodicity in the correlograms than real landscapes because all variation in the data is related to spatial patterning, and because our sampling point density is much higher than commonly feasible in field surveys. Random variation and sampling error will cloud periodicity in the analysis of real landscapes. However, regular patterns are not uncommon in the natural world and occur, for example, in the case of drumlin fields (Kabrick et al. 1997), sand dune ridges (Lichter 1998), spacing of animal territories (Hasegawa and Tanemura 1976), shrub and tree locations (Beals 1968, Kenkel et al. 1989), and the spatial spread of disturbances (Iwasa et al. 1991, Jeltsch and Wissel 1994). Spatial patterns occur also in population dynamics, such as travelling waves in vole populations (Ranta and Kaitala 1997). Koenig (1998, 1999) recently suggested analyzing changes in spatial autocorrelation of population data over time to foster understanding of spatial synchrony in population dynamics. Clearly, there appears to be great potential for both exciting research and incorrect conclusions when examining spatial periodicity using geostatistical models.

When working in real landscapes, the question of statistical significance arises because usually only a sample of the study area is available for analysis, and autocorrelation estimates may be spurious. Tests for statistical significance of correlograms have been developed (Clayton and Hudelson 1995) and are based on the number of pairs in a distance class. We did not employ such tests on our artificial data because our dataset was a complete population, rather than a sample, and because we could have increased the number of pairs in a distance class by increasing the size of our maps, thereby achieving statistical significance at any desired confidence level. However, when working with real data the number of sampling points will be commonly much smaller and statistical significance tests need to be applied. Furthermore, the specific research question will determine if statistical significance translates into ecological significance.

Our results and previous work suggest that autocorrelation between patches occurs and that it can cause periodicity in geostatistical models when spatial periodicity is present. This poses a problem for interpolation techniques such as kriging that are based on models fitted through variograms. Little attention has been paid to models that can capture periodicity adequately. So far, only sinusoidal models have been suggested (Cressie 1993, Lobo et al. 1998), but our study indicates that these are not always suitable.

The current confusion about the interpretation of periodicity in geostatistical models may be partly due to the inconsistent nomenclature in the literature. We suggest using the terms "periodicity in the correlo-

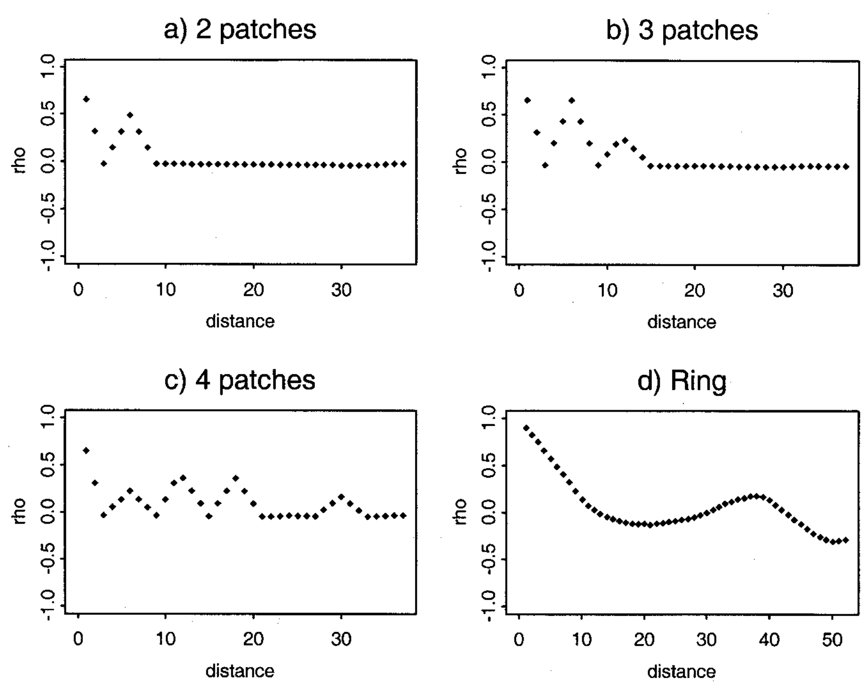

Fig. 8. Period correlograms resulting from spatial patterns that are not regular (Fig. 2), but were suspected to result in periodicity in the correlograms: a) 2 patches, b) 3 patches, c) 4 patches, and d) ring.

gram" (or semi-variogram), and correspondingly "spatial periodicity" in the data when identifying such patterns. The shape of the geostatistical model should be described using terminology commonly used for waves, i.e., "amplitude" for the vertical difference between "peaks" and "troughs", and "wavelength" for the distance between two peaks.

We do not advocate correlogram or semi-variogram analysis as the sole method for use in analyzing spatial pattern. Based on correlograms alone, it can be difficult to describe complex spatial patterns correctly because different spatial patterns can result in similarly-shaped correlograms. Furthermore, data sets capturing only a few patches are also prone to exhibit wave-like correlograms. The objective of this study was to explore the relationship between periodicity in landscape patterns and geostatistical models. The concurrent trends of increasing use of geostatistics in ecology and of rising interest in landscape ecology and spatial phenomena make it likely that many studies will encounter periodic spatial pattern. When interpreted carefully, periodicity in geostatistical models can provide a tool to detect and describe spatial periodicity.

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